Noise Output and Noise Figure of Biased Millimeter-Wave Detector Diodes*

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Summary—**The behavior of a dc biased millimeter-wave detector diode was investigated by theoretical analysis and experimental measurement. The results indicate that because of the nonlinearity of the diode, shot noise appearing across the diode increases with dc biasing. For the same reason conversion gain of the detector increases with bias. The increase in gain is faster than the increase in noise for a certain range of bias current. Thus the noise figure of the diode detector and its minimum detectable signal are decreased.**

Introduction

IT IS RECOGNIZED in radio engineering that a proper control of dc bias current[1-3] or rectified dc current[4] can optimize the sensitivity and noise of a crystal detector. Recently it has been found experimentally that the application of dc bias to a millimeter-wave detector (1N53) produces several interesting effects.[5] Among these are an increase in detection efficiency (gain) and an increase in sensitivity (smaller minimum detectable signal). In addition the noise figure of the system was reduced.

The major problem considered was this: It is commonly accepted that the noise generated in a semiconductor diode increases with increasing forward bias current.[1,2] If this is true, then how can the noise figure be decreased by doing something which increases noise in the system? This paper was written in an attempt to explain these results.

Noise Output

In this discussion, thermal noise due to the spreading resistance and shot noise across the differential resistance (a nonlinear resistance of the barrier) of the crystal detector are considered. Consideration of other noise such as current fluctuation noise or microphonic noise is omitted. According to Torrey and Whitmer,[2] the total noise power due to shot noise and thermal noise in a crystal detector is given by

$$P = \frac{RP_B + rkT\Delta f}{R + r}, \tag{1}$$

Δf = the noise bandwidth of the crystal detector (cps),
T = the temperature of the crystal (°K),
k = the Boltzmann's constant (Joule/°K),
r = the spreading resistance of the crystal detector (ohms),
R = the differential resistance of the crystal barrier (ohms), and
P_B = the shot noise for the crystal diode (watts).

The net current of the crystal diode is

$$I' = A(\epsilon^{e\beta V/kT} - \epsilon^{-e(1-\beta)V/kT}) = A\epsilon^{e\beta V/kT}(1 - \epsilon^{-eV/kT}), \tag{2}$$

where

A = a constant depending on the crystal (amperes),
V = the voltage applied to the barrier (volts),
e = the electron charge (coulombs), and
β = a parameter determining the amount of barrier lowering resulting from image force and tunnel effect.

In (2), $\epsilon^{-eV/kT}$ is negligibly small in comparison with unity for voltage larger than about 0.06 volt. Thus, for simplicity, it is assumed that

$$I' \approx A\epsilon^{e\beta V/kT}. \tag{3}$$

Provided that A is small, this equation will also apply for V less than 0.06 volt. The size of A does not appear to affect the percentage error for the approximation in (3), but it is helpful to keep the absolute error within a practically permissible limit if A is made small enough (for example, see Fig. 1).

It is almost impossible to measure the voltage directly across the barrier of a 1N53 but it is possible to measure the voltage across the diode. Also, (2) and (3) are derived under certain idealized conditions such as parallel uniform contacts with no fringing effect. For practical crystal diodes the situation may differ from the ideal. Thus it is assumed that the voltage across the barrier, V, is related to the voltage across the diode, V_0, by

$$V = aV_0. \tag{4}$$

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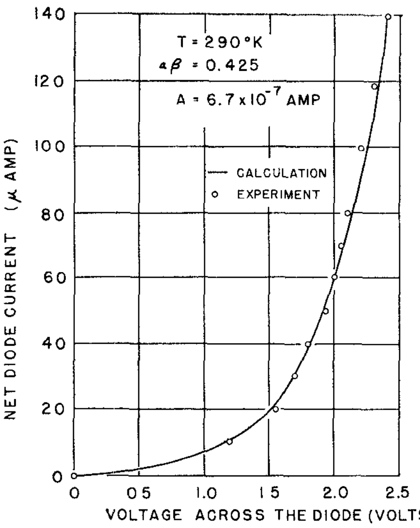

Fig. 1—Voltage-current characteristic of a 1N53 crystal diode.

Substituting (4) into (3),

$$I' \approx A\epsilon^{e\beta a V_0/kT}. \tag{5}$$

This assumption was examined experimentally. In Fig. 1 the net current, I', is plotted against V_0, the voltage across a 1N53 diode. The solid curve is the result calculated according to (5). For exact treatment parameters β and a might be functions of V_0. The results shown in Fig. 1 indicate that it is permissible to consider these parameters as constant, for simplicity. The results in Fig. 1 also indicate that, practically, it is permissible to use (5) to represent the 1N53 diode characteristic for positive voltages less than 2.5 volts.

The differential resistance of the diode is therefore

$$R = \left(\frac{dI'}{dV}\right)^{-1} = \left(\frac{dI'}{dV_0}\cdot\frac{dV_0}{dV}\right)^{-1}$$
$$= \left(A\,\frac{e\beta}{kT}\,\epsilon^{ea\beta V_0/kT}\right)^{-1} = \left(\frac{e\beta}{kT}\cdot I'\right)^{-1}. \tag{6}$$

Shot noise P_B is defined[2] as

$$P_B = \tfrac{1}{2}eIR\Delta f, \tag{7}$$

where I is the total current across the barrier

$$I = A(\epsilon^{e\beta V/kT} + \epsilon^{-e(1-\beta)V/kT})$$
$$= A\epsilon^{e\beta V/kT}(1 + \epsilon^{-(eV/kT)}). \tag{8}$$

Thus as in (3)

$$I \approx A\epsilon^{ea\beta V_0/kT} \approx I' \tag{9}$$

and

$$P_B = \tfrac{1}{2}\cdot\frac{kT\Delta f}{\beta}. \tag{10}$$

This indicates that the voltage-dependence of shot noise is insignificant. As Torrey and Whitmer stated, if β should decrease slowly with increasing voltage, P_B will increase with V_0 slowly. Our results indicated that the voltage-dependence of β was insignificant and that the noise rose very rapidly with increasing voltage. This rapid increase in noise can be explained by substituting (6) and (10) into (1). The total noise in the diode is now

$$P = \frac{\left(A\,\frac{e\beta}{kT}\,\epsilon^{ea\beta V_0/kT}\right)^{-1}\cdot\frac{1}{2}\cdot\frac{kT\Delta f}{\beta} + rkT\Delta f}{\left(A\,\frac{e\beta}{kT}\,\epsilon^{ea\beta V_0/kT}\right)^{-1} + r}. \tag{11}$$

This indicates that the voltage-dependence of the noise is due *mainly to the voltage-dependence of the differential resistance* R, and that the effect of the voltage-dependence of shot noise and other noise is probably minor.

Substituting (5) into (11),

$$P = \frac{\frac{1}{2}\left(\frac{kT}{\beta}\right)^2\frac{\Delta f}{e} + rkT\Delta f I'}{\frac{kT}{e\beta} + rI'}. \tag{12}$$

In (12) the noise P is expressed as a function of the net current I'. It is not easy to measure P exactly in practice because P is usually very small. However, the relative noise power level P_0 as a function of the net current I_0' can be easily observed.

$$P_0 = b\cdot\Delta f\,\frac{\frac{1}{2e}\left(\frac{kT}{\beta}\right)^2 + rkTI'}{\frac{kT}{e\beta} + rI'} \tag{13}$$

where

$$b \equiv P_0/P. \tag{14}$$

For $I'=0$,

$$P_{00} = b\cdot\Delta f\cdot\frac{kT}{2\beta}. \tag{15}$$

The relative noise power expressed in db as a function of the net current I' is

$$P_0(\text{db}) \equiv 10\log\frac{P_0}{P_{00}} = 10\log\frac{1 + \frac{2a'e\beta^2 r}{kT}I'}{1 + \frac{b'e\beta r}{kT}I'}. \tag{16}$$

Since no information about accurate theoretical value of β and r of the 1N53 is available at present, constants a' and b' are introduced. Actual β and r are probably not identical to the ones derived under an ideal assumption. The solid curve in Fig. 2 shows the result of calculations

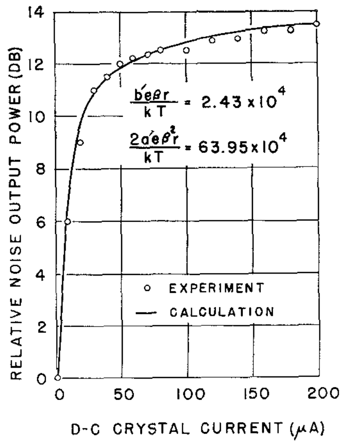

Fig. 2—Relative noise output vs dc crystal current with no input signal.

according to (16) with parameters properly adjusted to fit the experimental results. The experimental data are plotted in the same figure. The calculation and the experiment indicate that the noise in the detector diode truly increases with increasing crystal current. Thus some explanation other than a decrease in noise generated must be found for the decrease in noise figure with the application of bias.

Gain of the Diode

If the 1N53 is *assumed* to be a square-law detector, the crystal current, I', due to the dc bias voltage, V_0, and a small input microwave voltage, v, is given by a Taylor expansion,[2]

$$I' = f(V + v) \approx f(V_0) + \frac{df}{dV_0} v + \frac{1}{2} \frac{d^2f}{dV_0^2} v^2. \quad (17)$$

In this case, by (5),

$$\begin{aligned} f(V + v) &\approx A\epsilon^{ea\beta(V+v)/kT} \\ &\approx A\left\{1 + \frac{ea\beta(V+v)}{kT} + \frac{1}{2}\left[\frac{ea\beta(V+v)}{kT}\right]^2 \right. \\ &\quad \left. + \frac{1}{6}\left[\frac{ea\beta(V+v)}{kT}\right]^3 + \cdots \right\}. \end{aligned} \quad (18)$$

If the microwave resistance of the diode is R_{rf}, the input microwave power to the diode is

$$P_i = \frac{1}{2} \frac{v^2}{R_{rf}} \cdot \quad (19)$$

If the detected signal output resistance of the diode is R_0, the detected output power is

$$P_0 = \frac{1}{2} i_0^2 R_0, \quad (20)$$

where i_0 is the detected current:

$$i_0 = \frac{1}{2} \frac{d^2f}{dV_0^2} v^2. \quad (21)$$

Thus the conversion gain is given by

$$G = \frac{P_0}{P_i} = \frac{1}{4} R_0 v^2 R_{rf} \left(\frac{d^2f}{dV_0^2}\right)^2. \quad (22)$$

From (19)

$$v^2 = 2R_{rf}P_i. \quad (23)$$

Substituting (23) into (22)

$$G = \frac{1}{2} P_i R_0 R_{rf}^2 \left(\frac{d^2f}{dV_0^2}\right)^2. \quad (24)$$

If the microwave resistance of the diode R_{rf} is assumed to be proportional to the differential resistance

$$R = \left(\frac{df}{dV_0}\right)^{-1},$$

then

$$R_{rf} = \xi \left(\frac{df}{dV_0}\right)^{-1} \quad (25)$$

where ξ is a proportionality constant. Substituting (25) into (24)

$$G = \frac{1}{2} P_i \xi^2 R_0 \left[\frac{\dfrac{d^2f}{dV_0^2}}{\dfrac{df}{dV_0}}\right]^2. \quad (26)$$

If G_0 is the conversion gain for zero bias ($V_0 = 0$),

$$G_0 = \frac{1}{2} P_i \xi^2 R_0 \left[\frac{\left.\dfrac{d^2f}{dV_0^2}\right|_{V_0=0}}{\left.\dfrac{df}{dV_0}\right|_{V_0=0}}\right]^2. \quad (27)$$

The relative gain G(db) with G_0 as zero db is then

$$G(\text{db}) \equiv 10 \log \frac{G}{G_0} = 20 \log \left\{\frac{\dfrac{d^2f}{dV_0^2}}{\dfrac{df}{dV_0}} \cdot \frac{\left.\dfrac{df}{dV_0}\right|_{V_0=0}}{\left.\dfrac{d^2f}{dV_0^2}\right|_{V_0=0}}\right\}. \quad (28)$$

From (18)

$$\frac{\dfrac{d^2f}{dV_0^2}}{\dfrac{df}{dV_0}} = \frac{1 + \left(\dfrac{ea\beta}{kT}\right)^2 V_0}{1 + V_0 + \dfrac{1}{2}\left(\dfrac{ea\beta}{kT}\right)^2 V_0^2}$$

$$\approx \frac{1 + \left(\dfrac{ea\beta}{kT}\right)^2 V_0}{1 + V_0} \cdot \quad (29)$$

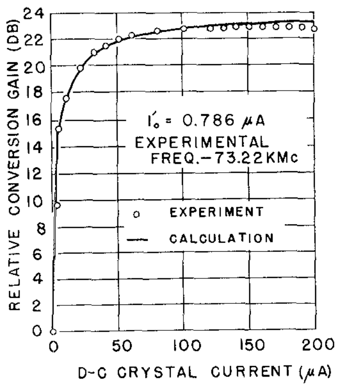

Fig. 3—Increase in relative gain caused by dc bias.

From (5)

$$V_0 = \frac{kT}{e\alpha\beta} \ln \frac{I'}{A} \cdot \tag{30}$$

Substituting (29) into (28) with (30),

$$G(\text{db}) = 20 \log \frac{1 + \dfrac{e\alpha\beta}{kT} \ln \dfrac{I'}{A}}{1 + \dfrac{kT}{e\alpha\beta} \ln \dfrac{I'}{A}} + 20 \log \frac{1 + \dfrac{kT}{e\alpha\beta} \ln \dfrac{I_0'}{A}}{1 + \dfrac{e\alpha\beta}{kT} \ln \dfrac{I_0'}{A}} \cdot \tag{31}$$

In (31) I_0' is the detected current when the dc bias is zero. The solid curve in Fig. 3 is the calculated gain curve using the same parameter values employed in Fig. 1 and with $I_0' = 0.786$ μa. The results show that the theoretical equation (31) does predict the gain increase and that the gain of the 1N53 diode really does increase rapidly with increasing bias. It is interesting to note that *the increase in gain with dc bias is much faster than the increase in noise* shown in Fig. 2.

Noise Figure of Biased Diode

By definition, the noise figure of a dc biased detector can be expressed by the following equation.[6]

$$F = \frac{P}{N_i G} \tag{32}$$

where

P = the available noise output of the diode,
N_i = the available noise input to the diode, and
G = the gain of the diode.

[6] M. Schwartz, "Information Transmission, Modulation and Noise," McGraw-Hill Book Co., Inc., New York, N. Y.; 1959.

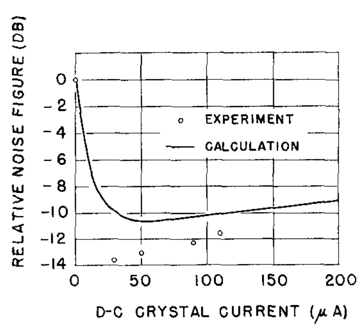

Fig. 4—Noise figure of biased 1N53 detector relative to zero bias.

If P_0 represents the available noise output of the diode for zero bias and G_0 represents the gain of the diode for zero bias, then the noise figure for zero bias is

$$F_0 = \frac{P_0}{N_i G_0} \cdot \tag{33}$$

The relative noise figure of the biased diode F(db), with F_0 as zero db, is

$$F(\text{db}) \equiv 10 \log \frac{F}{F_0} = 10 \log \frac{PG_0}{GP_0} = P(\text{db}) - G(\text{db}) \tag{34}$$

In (34) the relative noise output P(db) is given by (16) and the relative gain G(db) is given by (31).

The calculated result of (34) is shown in Fig. 4. The experimental data of the relative noise figure of the 1N53 diode is also plotted in the same figure. The noise figure of a 1N53 diode in a DBB119 crystal mount followed by a 441B amplifier showed an over-all noise figure of 40 db at zero dc bias on the diode at 73.22 kMc. The calculated and experimental results show that the noise figure decreases when the diode is properly biased. Theoretical equation (34) indicates that *the gain increase due to the dc bias is much faster than the increase in noise power*. This is why the noise figure decreases with dc bias in spite of the rising noise output.

Minimum Detectable Signal (MDS) of Biased Diode

If the minimum detectable signal (MDS) of the detector diode S is defined as the input signal power which makes signal output power twice the noise output, it is known that

$$S \equiv FN_i \tag{35}$$

where

S = the MDS of the diode,
F = the noise figure of the diode, and
N_i = the noise input to the diode.

When the crystal diode is dc biased, as was explained in the previous section, the noise figure F changes. If

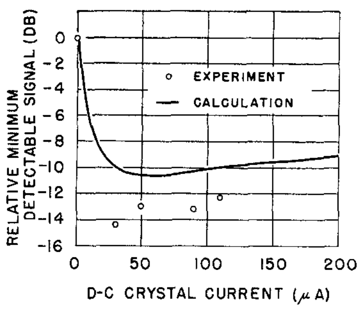

Fig. 5—Minimum detectable signal of 1N53 detector relative to zero bias.

the MDS at zero dc bias is S_0,

$$S_0 = F_0 N_i \tag{36}$$

where F_0 is the noise figure of the diode when no dc bias is applied. Thus the relative MDS S(db) can be expressed by

$$S(\text{db}) \equiv 10 \log \frac{S}{S_0} = 10 \log \frac{F}{F_0}$$

$$= F(\text{db}) = P(\text{db}) - G(\text{db}). \tag{37}$$

In Fig. 5 the calculated relative MDS and experimentally obtained relative MDS are plotted with zero bias MDS as zero db. The zero dc bias MDS of a 1N53 diode in a DBB119 crystal mount followed by a 441B amplifier was −50.8 dbm at 73.22 kMc. Experiment and calculation show that the MDS of the diode decreases with increasing dc bias current up to approximately 70 μa.

Conclusions

The conclusions which have been reached by theoretical analysis and experimental investigation are as follows:

1) Biasing the detector increases the noise generated in the crystal;
2) This increase in noise is small when compared with the increase in gain;
3) Thus, even though noise increases, the noise figure of a device can decrease.

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Microwave Variable Attenuators and Modulators Using *PIN* Diodes*

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Summary—The *PIN* diode is a double diffused junction with an intrinsic layer separating the *P* and *N* regions. At frequencies above 100 Mc, the diode ceases to be a rectifier because of carrier storage and transit time effects. Its shunt capacitance is quite small because of the separation of the *P* and *N* regions by the *I* layer. Conductivity of the *I* region can be varied by a dc bias current and the device becomes an electrically variable resistor which can be used for microwave attenuators and modulators up to frequencies as high as 20 Gc.

The *PIN* junctions are mounted on posts which are inserted in a 50-ohm strip transmission line as shunt elements, and a number of these elements, spaced a quarter wavelength apart at midband, are used to form an attenuator. At the appropriate bias current, yielding 50-ohm junction resistances, the diode elements are reactively compensated by choice of post dimensions so that they are effectively pure resistances, yielding an image attenuation of 4.2 db per element. Many elements can be used to attain any desired total attenuation and higher impedance end elements can be used to improve the SWR. Bandwidths of 4 to 1 with low SWR in both ON and OFF conditions are achievable. Maximum attenuation of 60 db, insertion loss of 1 db, and SWR of 1.5 are typical for a 12-diode attenuator and powers of the order of watts can be handled with negligible harmonic generation. When used as a pulse modulator, rise times of the order of 10 nsec are achievable.

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